Uncertain Length of Life, Retirement Age, and Optimal Pension Design*

Thomas Aronsson[α] and Sören Blomquist[β]

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Abstract
In this paper, we consider how the hours of work and retirement age ought to respond to a change in the uncertainty of the length of life. In a first best framework, where a benevolent government exercises perfect control over the individuals' labor supply and retirement-decisions, the results show that a decrease in the standard deviation of life-length leads to an increase in the optimal retirement age and a decrease in the hours of work per period spent working. This result is robust, and is also derived in models of decentralized decision-making where individuals decide on their own consumption, labor supply, and retirement age, and where the government attempts to affect their behavior and welfare through redistribution and pension policy.

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[α] Address: Department of Economics, Umeå School of Business and Economics, Umeå University, SE – 901 87 Umeå, Sweden.
E-mail: Thomas.Aronsson@umu.se.
[β] Address: Department of Economics, Uppsala University, SE – 751 20 Uppsala, Sweden.
E-mail: Soren.Blomquist@nek.uu.se.

1. Introduction

It is well known that the average length of life has increased considerably in the western world during the last 150 years and more recently also in less developed countries. Partly in response to this development, there is now a large literature dealing with the ageing of the population and the consequences this ought to have for the design of pension schemes.[1] Another important change in the mortality process, which has gained much less attention in the economics literature, is that the *standard deviation* of the length of life has decreased. Sweden has unusually good historical demographic statistics, so it is possible to follow this development over time. As the mortality among infants has a large influence both on the expected length of life and on the standard deviation, it is customary to calculate the standard deviation (or variance) in the length of life for those who have survived until at least age 10. This measure is usually denoted s_{10}. For Sweden, s_{10} was 21 around 1750 and had decreased to around 12.5 in year 2000.[2] There is also a large cross-country variation in the standard deviation of life-length. This is particularly so if one compares developed and less developed countries, although there is variation also between developed countries. For instance, while Sweden has the most equal distribution of life-length with s_{10} being around 12.5 years, the U.S. has one of the most unequal distributions among developed countries with a standard deviation, s_{10}, of 15 years.[3]

How would the optimal retirement age, annual hours of work when working, the total labor supply over the life-cycle, as well as the individual contribution to and benefit from the pension system, respond to changes in the standard deviation of the length of life? This, of course, depends on what we see as the reason for having a public pension scheme. Many alternative motives for public pension schemes have been discussed over time. One is that the market for annuities is not well functioning. It is, therefore, difficult for individuals to handle the uncertainty of the length of life. How much should be saved for the old age? How should individuals plan their consumption path in the old age? One role for a public pension scheme is to mitigate the problems associated with an uncertain length of life. It is this property that we focus on in the present paper. Our analysis shows that the standard deviation of the length

[1] This literature focuses on a variety of issues such as how the pension system affects retirement incentives at the individual level and labor supply behavior among the elderly (e.g., Crawford and Lilien, 1981; Gruber and Wise, 1999; Coile and Gruber, 2007), the optimal legal retirement age (Lacomba and Lagos, 2006) as well as the direction of reforms of social security (Diamond, 2005; Diamond and Orszag, 2005).

[2] See Edwards and Tuljapurkar (2005, p. 654).

[3] See Edwards and Tuljapurkar (2005, p. 653).

of life has important implications for the optimal retirement age. The nature of this influence depends crucially on individuals' risk aversion with respect to the number of years they plan to spend in retirement.[4]

In a seminal contribution to the literature on social security and retirement, Crawford and Lilien (1981) examine how the US social security system affects the retirement decision. Since our analysis is based on a modified version of their model, it can be of interest to shortly describe their study. Under the assumptions of perfect capital markets, actuarial fairness, and certain lifetimes, they show that the US social security system has no effect on the retirement decision. The intuition is that the forced pension system savings are completely undone by decreased private savings. Yet, by relaxing each one of these assumptions, they find that the social security system does affect the retirement decision. In particular, they provide a detailed analysis of how the fact that the length of life is uncertain affects how the social security system influences the retirement decision. We will use a slightly more general version of their model, but study a completely different issue; namely how changes in the standard deviation of the length of life affect the *optimal design* of a public pension system. As such, we pay special attention to the effects of changes in the uncertainty of the length of life.

To obtain a tractable model, we make several simplifying assumptions, the details of which will be laid out in section 2 below. One important simplification is that the individuals in our model have no better knowledge about their own mortality than the policy maker (i.e., we abstract from asymmetric information).[5] Another is that we disregard bequests by focusing on a single generation. We also assume that the objective of the policy maker is to design the pension scheme in such a way that the expected lifetime utility faced by the representative consumer is maximized.[6] Part of the solution to this problem is obtained by providing a certain consumption stream during the old age independently of how old an individual becomes. We shall both consider a first best framework where the government exercises full

[4] There are studies focusing on other aspects of the variation in the length of life between individuals. For example, Bommier et al. (2007) consider redistribution between individuals with different life-lengths. A crucial assumption in their work is that the individual life-time utilities exhibit temporal risk aversion.

[5] We also abstract from other sources of asymmetric information, such as unobserved differences in the ability to work during old age. Cremer et al. (2004) consider a model where the productivity and health status are private information and vary between consumers. They show that the second best optimal policy may imply a distortion of the retirement behavior.

[6] There are alternative ways to formulate the objective of the policy maker. For instance, the objective might be to maximize some function or individuals' realized lifetime utilities. See Fleurbaey et al. (2016).

control over the resource allocation by deciding upon individual consumption, labor supply, and retirement ages, as well as three different versions of a model with decentralized decision-making. The first decentralized framework is a laissez faire economy, where the resources left over at individuals' time of death are wasted. Albeit unrealistic in itself, this will provide a reference case for the subsequent analysis of policy intervention. In the second case, we assume there is a government that confiscates all the consumption left over when individuals die and then distribute these resources in an optimal way among those alive. However, as a confiscation of the resources individuals leave behind when dying is hardly a feasible policy tool, we also consider a version of the model with a pension system, where the government raises revenue by a proportional tax (pension fee) on labor income to finance a pension benefit per year spent in retirement. If the pension system is actuarially fair such that the life-time contribution is equal to the expected life-time benefit, a first best allocation can be achieved.

For all models considered, and under reasonable assumptions about the curvature of the function representing the utility derived from the years spent in retirement, a decrease in the standard deviation of the length of life leads to an increase in the retirement age. Also, an increase in the expected length of life implies that the retirement age should be increased. An interesting policy implication of our analysis is that Sweden and the US should have different retirement ages due to differences in the standard deviation of life-length, even if we were to disregard all other differences between the two countries. As far as we know, the connection between the standard deviation of life-length and the optimal retirement age has not been examined before.

The rest of this paper is organized as follows. In section 2, we present the model of individual behavior. Section 3 deals with the first best decision-problem and solution; in particular, we examine how the resource allocation responds to a change in the standard deviation of the length of life as well as a change in the average length of life. In section 4 we consider three models of decentralized decision making. Section 5 concludes.

2. The Model

To be able to focus on the problem at hand in the simplest possible way, we formulate the model such that the timing of consumption is not an issue. We also assume that the hours of work are the same during all years spent in the labor market. The important choices in our model are the retirement age and hours of work before retirement, respectively; not the tradeoff between consumption or work hours at different dates. Focusing on these decisions is also in agreement with much of the empirical labor supply literature, at least for men, where the hours of work when working do not vary much between years, while the timing of retirement is sensitive to economic incentives (Blundell et al. 2011).

Our study is based on a discrete and to some extent generalized version of the Crawford and Lilien (1981) model. As they do, we assume a zero rate of interest. To arrive at our preferred specification, we begin by briefly describing their model. Crawford and Lilien assume that the hours of work when working are fixed and exogenously given. They normalize the utility of leisure to zero during the work period of life and measure the annual utility of leisure during retirement by $V(l_t)=v$. In discrete form, we can write the utility function that Crawford and Lilien use as follows:

$$\sum_{t=1}^{T} U(C_t)+\sum_{t=R}^{T} V(l_t)=\sum_{t=1}^{T} U(C_t)+(T-R)v$$

where C_t denotes consumption and l_t leisure time in period t; R denotes the retirement age and T the length of life. We modify their utility specification in three ways. First, based on the arguments above, the utility of private consumption will be written as $\sum_{t=1}^{T}\min U(C_t)$. By assuming perfect capital markets, perfect smoothing ($C_t=C$ for all years) can thus be obtained.[7] Second, we introduce a choice of work hours during the pre-retirement period of life. Letting H denote the annual time endowment and h the hours of work, we write the utility of leisure during the working period as $\sum_{t=1}^{R}\min\Phi(H-h_t)=R\Phi(H-h)$. By a similar formulation, the utility of leisure during the retirement period is given by $\sum_{t=R}^{T}\min\Phi(H)=(T-R)\Phi(H)$. Combining these functions gives a possible formulation of life time utility as

$$\tilde{\Omega}=TU(C)+R\Phi(H-h)+(T-R)\Phi(H). \tag{1a}$$

[7] We could, of course, introduce borrowing constraints. However, as our model is constructed these constraints would not be binding.

The utility formulation in equation (1a) accords well with a large part of the retirement literature.[8] However, since our focus is on the retirement decision, it seems overly restrictive for at least two reasons. One is that the function measuring the utility of annual leisure time is the same during the working and retirement years. Another is that the marginal utility of a year spent in retirement is constant. In our view, these features of equation (1a) are not realistic.

The step from being a worker to being retired opens up a whole new spectrum of opportunities regarding time-use (e.g., developing new, and time-consuming, hobbies) as well as implying fewer restrictions on the residential choice. As a consequence, the utility associated with retirement may differ in a fundamental way from the utility of leisure during the working-life, which motivates that these two aspects of "non-working time" are treated separately. Also, it is clearly plausible that the marginal utility of the first retirement year is different from, say, the thirtieth year of retirement. Our third modification of the Crawford-Lilien utility function is therefore to make a concave transformation $\Pi\left(\left(T-R\right)\Phi\left(H\right)\right)$ of the retirement component. For notational convenience we condense this to $P\left(T-R\right)$ with $P'>0, P''<0$, i.e, the marginal utility of retirement years is decreasing in the number of years in retirement. The properties of the function $P\left(\cdot\right)$ drive many of our results; in particular, the sign of the third derivative will determine the sign of important comparative statics. Introducing the concave transformation is crucial for the main results derived below: while the functional form given by equation (1a) implies that uncertainty about the length of life is of no consequence for the retirement behavior, the concave transformation implies that uncertainty becomes important. The formulation of the utility function that we use then becomes

$$\Omega = TU\left(C\right) + R\Phi\left(H-h\right) + P\left(T-R\right). \tag{1b}$$

We assume that the length of life is a random variable in the sense that $T = \bar{T} + \gamma\varepsilon$, where $\bar{T}$ is the expected life-length, ε a random variable with mean zero and unit variance, and $\gamma > 0$ a parameter. This means that γ is interpretable as the standard deviation of the length of life.

[8] Note that this formulation also allows for the possibility of fixed costs of work. For instance, the function Φ could take the form $\Phi\left(H-h\right) = \phi\left(H-h\right) - \eta\ \mathbf{1}\left(h>0\right)$, but there could also be other differences between Φ and ϕ.

In the absence of any pension system, and since we assume a zero interest rate, the life-time budget constraint facing the individual can be written as

$$\sum_{t=1}^{R} w_t h_t - \sum_{t=1}^{T} C_t = 0 \tag{2}$$

where w is the hourly gross wage rate. In the next two sections, we use this basic model to analyze relationships between, on the one hand, the optimal retirement age and pension design and, on the other, the standard deviation of the length of life.

3. A First Best Approach

We start by considering a first best decision-problem, where the policy maker decides upon the consumption, hours of work, and retirement age for a large number of ex ante identical individuals.

Since the age of death is a stochastic variable, the budget constraint facing an individual will be stochastic as well. However, we assume that the number of individuals in the economy as a whole is large enough to imply that the resource constraint at the aggregate level can be treated as deterministic. The resource constraint for the policy maker can, therefore, be written as

$$\bar{T}C - Rwh = 0\,. \tag{3}$$

The objective of the policy maker is to maximize the expected utility of a typical individual, i.e.,

$$E[TU(C) + R\Phi(H-h) + P(T-R)] = \bar{T}U(C) + R\Phi(H-h) + E[P(T-R)] \tag{4}$$

where E denotes the expectations operator. Using $T = \bar{T} + \gamma\varepsilon$, where $E(\varepsilon) = 0$ and $Var(\varepsilon) = 1$, and then substituting equation (3) into equation (4), we can write the optimization problem of the policy maker as

$$\underset{R,h}{Max}\ \bar{T}U\left(\frac{Rwh}{\bar{T}}\right) + R\,\Phi(H-h) + E\left[P(T-R)\right].$$

The first order conditions for R and h become

$$U'(C)wh + \Phi(H-h) - E\left[P'(T-R)\right] = 0 \tag{5a}$$

$$U'(C)w - \Phi'(H-h) = 0\,. \tag{5b}$$

We can then derive the following functions for the optimal retirement age and hours of work;

$$R = R(w, \overset{+}{\bar{T}}, \gamma) \tag{6}$$

$$h = h(w, \overset{-}{\bar{T}}, \gamma) \tag{7}$$

in which the sign of the comparative statics derivative (when the sign is unambiguous) is given above each argument. The comparative statics derivatives are presented in the Appendix. In general, an increase in the hourly wage rate may either lead to increased or decreased labor supply (measured both in terms of *h* and *R*), depending on whether the substitution effect dominates the income effect, or vice versa. In the special case with a quasi-linear utility function, i.e., where the utility function is linear in *C*, it is straightforward to show that an increase in the wage rate leads to an increase in the retirement age <u>and</u> an increase in the hours of work per period when participating in the labor market: in other words, the labor supply would increase in both dimensions.

Let us now turn to how changes in the length of life parameters $\bar{T}$ and γ affect the optimal retirement age and hours of work prior to retirement, respectively. As indicated by equations (6) and (7), an increase in the average length of life, $\bar{T}$, leads to an increase in the retirement age and a decrease in the number of work hours per period prior to retirement. This result is intuitive: if individuals are expected to live longer, ceteris paribus, it is first best optimal to extend the number of periods of labor market participation and thus increase the retirement age. In turn, the higher income (due to additional periods of work) leads to a decrease in the marginal utility of consumption, which provides an incentive to reduce the optimal number of work hours per period before retirement. The net change in work hours over the whole life-cycle is, nevertheless, positive such that $\partial(Rh)/\partial\bar{T} > 0$, meaning that an increase in the life-expectancy leads to an increase in the life-time labor supply.

The qualitative effects of an increase in the standard deviation of life-length, γ, are ambiguous in general. One can show that $sign\; \partial R/\partial\gamma = -sign\; \mathrm{cov}(\varepsilon, P'')$ and $sign\; \partial h/\partial\gamma = sign\; \mathrm{cov}(\varepsilon, P'')$. A sufficient condition for $\mathrm{cov}(\varepsilon, P'')$ to be positive is that the sub-utility function capturing the preferences for the number of years spent in retirement, $P(\cdot)$, is characterized by constant or decreasing absolute risk aversion in the sense that

$$-\frac{P''(X_2)}{P'(X_2)} \leq -\frac{P''(X_1)}{P'(X_1)} \text{ for } X_2 > X_1.$$

With increasing absolute risk aversion, on the other hand, $\mathrm{cov}(\varepsilon, P'')$ can be either positive or negative. To give some intuition as to why non-increasing absolute risk aversion might be plausible in this case, let $X=T-R$ denote the number of years in retirement. Also, let X_0 denote a certain number of retirement years and $\tilde{x}$ a mean zero lottery of retirement years. The risk premium, π, can then be defined as the number of certain retirement years a person is willing to forego to avoid the lottery, i.e., $\pi : EP\left(X_0+\tilde{x}\right)=P\left(X_0-\pi\right)$. Under decreasing (constant) absolute risk aversion, the risk premium is decreasing (constant) in X_0. In other words, the individual is at least not willing to pay less for avoiding a small lottery of retirement years when the number of certain years in retirement decreases.

Therefore, if $\text{cov}(\varepsilon, P")>0$, then $\partial R/\partial\gamma<0$ and $\partial h/\partial\gamma>0$. The interpretation follows naturally from what was said above. If the standard deviation of the length of life increases, it is optimal to retire earlier in order to be able to enjoy the equivalent of the same number of certain years in retirement as before. Phrased in a slightly different way, the optimal response will be to decrease the retirement age, which increases the likelihood that any consumer will be able to enjoy a period of retirement. Conversely, reduced uncertainty about the length of life, i.e., a decrease in γ, gives an analogous incentive to postpone the retirement, because the future benefits associated with retirement are less uncertain than before. Note also that earlier retirement causes the life-time income to decrease, ceteris paribus, which, in turn, increases the number of work hours per period before retirement through an income effect. The total number of work hours measured over the whole life-cycle will, nevertheless, decrease when the standard deviation of life-length increases, i.e., $\partial(Rh)/\partial\gamma<0$, meaning that the effect on h is not strong enough to fully offset the decrease in the total labor supply that increased uncertainty gives rise to.

In summary, we have derived the following results:

Proposition 1. *In the first best, an increase in the life-expectancy, $\bar{T}$, leads to an increase in the optimal retirement age and a decrease in the hours of work per period before retirement. The total number of work hours measured over the whole life-cycle increases. If the preferences for the number of years spent in retirement are characterized by constant or decreasing absolute risk aversion, a decrease in the standard deviation of the length of life, γ, leads to an increase in the optimal retirement age, a decrease in the hours of work per period before retirement, and an increase in the total number of work hours measured over the whole life-cycle.*

4. Decentralized Resource Allocations

In this section, we examine a decentralized setting in which each consumer decides on his/her own consumption, work hours, and retirement age. Our concern is also here to analyze how changes in the life-expectancy, $\bar{T}$, and the standard deviation of the length of life, γ, affect the hours of work and retirement age. At least three versions of such an economy come to mind. First, a laissez faire economy in which there is no public sector involvement at all; we consider this scenario as a point of reference. Second, a case where the resources left-over, due to that people may die earlier than they have planned for, are confiscated by the government and returned to the economy in the form of a per-person transfer to those alive. Yet, since such a confiscation is hardly a politically feasible alternative, we also consider a third case where the government implements an actuarially fair pension system. We address each of these cases in turn.

In reality, the consumption of an individual may vary over the life-cycle; for instance, the planned consumption might be smaller for years where the probability of living is low than for years where the probability of living is high. Likewise, the hours of work might vary between periods of the working life. To be able to focus attention on retirement behavior and pension-policy in a simple way, we use the model developed in Section 2 and examined in a first best setting in Section 3, where the individual has a maximin utility function. The individuals thus plans for the consumption per period to be constant over the whole life-cycle and the hours of work per period before retirement to be constant as well. The utility function facing any individual *i* takes the same form as equation (1b), i.e.,

$$\Omega^i = T^i U\left(C^i\right) + R\Phi\left(H - h^i\right) + P\left(T^i - R\right).$$

The individual plans for the eventuality of living until the age of $T^{\max}$ years, although the probability of this event might be quite small. This assumption is, of course, arbitrary. Our argument is that, if the individual does not consider the possibility of living very long, he/she may end up with too little resources during old age. As long as the individual plans for the possibility that his/her life may be longer than the average life-length, the final year assumed in the budget is not important. To simplify the analysis, we treat $T^{\max}$ as a constant in what follows.[9]

Laissez Faire Allocation

Without any public sector involvement at all, the individual budget constraint takes the form

$$R^i wh^i - T^{\max} C^i = 0. \tag{8}$$

Therefore, since our model abstracts from bequest motives, the resources left over if the individual does not live $T^{\max}$ years are wasted. The individual chooses consumption, work hours, and retirement age to maximize the expected utility subject to the budget constraint, i.e.,

$$\underset{C^i, h^i, R^i}{Max} \quad E[T^i U(C^i) + R^i \Phi(H - h^i) + P(T^i - R^i)] \tag{9}$$

subject to equation (8). Since T^i is stochastic, in all events except when $T^i = T^{\max}$ the individual will leave a "bequest". We assume this bequest gives no utility to the individual. It is a consequence of the uncertain life-time and the absence of a market for annuities. As before, we have $T^i = \bar{T} + \gamma\varepsilon^i$, where $E(\varepsilon^i) = 0$ and $Var(\varepsilon^i) = 1$. By substituting the budget constraint into the objective function, the decision-problem facing individual *i* can be written as

$$\underset{h^i, R^i}{Max}\ \bar{T}\, U\left(\frac{R^i wh^i}{T^{\max}}\right) + R^i \Phi(H - h^i) + E\left[P(T^i - R^i)\right].$$

Each consumer behaves as an atomistic agent and treats the before-tax wage rate as exogenous. The first order conditions for h^i and R^i become

$$\frac{\bar{T}}{T^{\max}} U'(C^i) w - \Phi'(H - h^i) = 0 \tag{10a}$$

$$\frac{\bar{T}}{T^{\max}} U'(C^i) wh^i + \Phi(H - h^i) - E[P'(T^i - R^i)] = 0. \tag{10b}$$

As the individuals are identical ex-ante, they will choose the same number of work hours and the same retirement age. The weight $\bar{T} / T^{\max}$ attached to the marginal utility of consumption appears as a consequence of expected utility maximization in combination with the assumption that the consumer recognizes that he/she may reach the age $T^{\max}$. This is also the

[9] Another alternative would be to assume that $\bar{T}$ and $T^{\max}$ are (in part) driven by the same underlying process. Although this change of assumption would influence the exact comparative statics with respect to $\bar{T}$ in the laisses fair equilibrium briefly discussed below, it has no other effect on the results and interpretations.

reason as to why the choices made by each individual do not satisfy the first order conditions of the social planner derived in the previous section. Equations (10a) and (10b) implicitly define the number of work hours and retirement age as follows:

$$R^i = \widehat{R}(w, \overline{T}, \gamma) \tag{11}$$

$$h^i = \widehat{h}(w, \overline{T}, \gamma) \tag{12}$$

where the dependence of h^i and R^i on the constant $T^{\max}$ has been suppressed. Again, since the individuals are identical ex-ante, we have $h^i = h$ and $R^i = R$ for all *i*.

Although the individuals acting in a laissez faire economy make choices other than those preferred by the social planner in Section 3, the effects of changes in the key parameters $\overline{T}$ and γ are, nevertheless, qualitatively similar to those presented in Proposition 1.[10] A decrease in the standard deviation of the length of life has the same behavioral implications here: it leads to an increase in the retirement age and a decrease in the hours of work per period spent in the labor force, if the preferences for the number of years spent in retirement are characterized by constant or decreasing absolute risk aversion. Similarly, the total number of work hours measured over the whole life-cycle, Rh, will increase in response to a lower γ. The only qualitative difference is that an increase in $\overline{T}$ may either lead to an increase or decrease in the hours of work per period spent in the labor force, while it leads to an unambiguous increase in the retirement age (as before). The possibility that h increases in response to a higher $\overline{T}$ is due to the multiplier $\overline{T}/T^{\max}$ in equations (10a) and (10b), which was not part of the corresponding first order conditions satisfied by the first best resource allocation given by equations (5a) and (5b). The intuition is that a higher $\overline{T}$ in this case gives rise to a substitution effect reminiscent to that of an increase in the wage rate. The total number of work hours measured over the whole life-cycle, Rh, increases in response to an increase in $\overline{T}$.

Redistribution Through Confiscation of the Resources Left over at Death

The decentralized resource allocation examined so far implies a waste of resources due to that individuals plan for the eventuality of living until the age of $T^{\max}$. This suggests a natural role for redistribution through surplus sharing. Suppose that the resources left over when people

[10] The derivation of these results is analogous to the derivation of the results in Proposition 1 and is, therefore, omitted.

die before the age of $T^{\max}$ are collected through a confiscatory inheritance tax and then redistributed to those alive. Since each individual plans for the possibility of living until the age of $T^{\max}$, the individual budget constraint changes to read

$$R^i wh^i + (T^{\max} - \bar{T})C = T^{\max} C^i \quad (13)$$

where $(T^{\max} - \bar{T})C$ represents the resources left over if an individual would die at age $\bar{T}$ instead of at age $T^{\max}$, which can then be transferred to another individual. This policy ensures that the overall resource constraint, $Rwh = \bar{T}C$, is satisfied.

Two possibilities arise here. First, the government may just announce $(T^{\max} - \bar{T})C$ as a lump-sum transfer. In this case, each individual will still satisfy the first order conditions given in equations (10a) and (10b), with the only modification that the number of work hours per period spent working as well as the retirement age will be lower than implied by equations (11) and (12) due to the income effect that this policy gives rise to. Therefore, the qualitative effects of an increase in γ and $\bar{T}$, respectively, will be the same as in the laissez faire allocation described above. Second, the government may announce the policy rule, i.e., each individual will receive a benefit proportional to his/her per-period consumption for each additional year of life beyond $\bar{T}$. In this case, the budget constraint given in equation (13) would effectively change to read

$$R^i wh^i - \bar{T}C^i = 0, \quad (14)$$

meaning that the decentralized allocation would be equivalent to the first best resource allocation analyzed in Section 3. In this case, therefore, the behavioral responses to changes in $\bar{T}$ and γ will be those described in Proposition 1.

Actuarially Fair Pension System

Suppose instead that each individual pays a pension fee, τ, proportional to income when active in the labor market and receives a pension benefit, B^i, per period spent in retirement. The budget constraint the individual uses for his planning then becomes

$$R^i wh^i (1-\tau) + (T^{\max} - R^i)B^i = T^{\max} C^i . \quad (15)$$

We will characterize an actuarially fair pension system such that the total contribution to the pension system equals the expected benefit, i.e.,

$$\tau R^i wh^i = (\bar{T} - R^i)B^i . \quad (16)$$

Solving equation (16) for $B^i = \tau R^i wh^i / (\bar{T} - R^i)$ and substituting into equation (15) gives

$$R^i wh^i + \frac{(T^{\max} - \bar{T})}{(\bar{T} - R^i)} \tau R^i wh^i = T^{\max} C^i \,. \tag{17}$$

The individual chooses consumption, work hours, and retirement age to maximize the expected utility in equation (9) subject to the budget constraint given in equation (17). As before, since the individuals are identical ex-ante, all of them will make the same choices such that $C^i = C$, $h^i = h$, and $R^i = R$ for all *i*.

Now, suppose that the government sets the pension fee, and implicitly also the pension benefit, to equalize the per period consumption over the individual life-cycle. This means

$$C = wh(1-\tau) = \frac{\tau R wh}{\bar{T} - R} = B \,, \tag{18}$$

implying $\tau = (\bar{T} - R)/\bar{T}$. Substituting this expression for τ into the individual budget constraint gives the resource constraint in equation (14). Therefore, if the pension fee is determined according to equation (18), the individuals will choose their consumption, work hours, and retirement age according to first best principles, in which case Proposition 1 applies.

How would the contribution rate to the pension system, τ, and the pension benefit per year spent in retirement, B, change in response to a variation in γ? To address this question, we assume (as we did above) that the preferences for the number of years spent in retirement are characterized by constant or decreasing absolute risk aversion. Note first that $\partial R / \partial \gamma < 0$ implies $\partial \tau / \partial \gamma = \partial [(\bar{T} - R)/\bar{T}] / \partial \gamma > 0$. Next, by using $\tau = (\bar{T} - R)/\bar{T}$ in equation (18), such that $B = (wRh)/\bar{T}$, and since $\partial (Rh)/\partial \gamma < 0$, we can derive $\partial B / \partial \gamma < 0$. Therefore, a decrease in the standard deviation of the length of life leads to a decrease in the contribution rate and an increase in the pension benefit per period spent in retirement, ceteris paribus. The intuition is that a decrease in γ leads to an increase in the life-time labor supply. In turn, the higher life-time income allows the government to respond through a simultaneous decrease in τ and increase in B. We summarize the most important results derived in Section 4 by Proposition 2.

Proposition 2. *Irrespective of whether the decentralized resource allocation is laissez faire or first best (through surplus sharing announced via a policy rule or an actuarially fair pension system), a decrease in the standard deviation of the length of life,* γ*, leads to (i) an increase in the retirement age,* R*, (ii) a decrease in the hours of work per period spent in the labor force,* h*, and (iii) an increase in the total number of work hours over the whole life-cycle,* Rh*, if the preferences for the number of years spent in retirement are characterized by constant or decreasing absolute risk aversion. Under an actuarially fair pension policy, and if the preferences for the number of years spent in retirement feature constant or decreasing absolute risk aversion, a decrease in* γ *will also lead to a decrease in the contribution rate (pension fee per unit of income) and an increase in the pension benefit per period spent in retirement. An increase in the life-expectancy,* $\bar{T}$*, leads to increases in* R *and* Rh*, respectively, while the effect on* h *is ambiguous unless the resource allocation is first best.*

5. Conclusion

Contrary to earlier studies on the relationships between life-length, retirement age, and public policy, which typically concentrate on effects of changes in the average length of life, the present paper focuses much attention on how the retirement age and labor supply ought to change in response to a change in the uncertainty about the length of life, as measured by the standard deviation of the length of life. Such a study has clear practical relevance both because this standard deviation has decreased substantially in most countries during the latest centuries, and because significant differences between countries still remain. Consequently, it is important to understand how changes in the standard deviation of life-length affect the retirement behavior, and how we can design the intertemporal redistribution system to achieve a socially optimal resource allocation. The present paper serves this purpose.

Our results show that the attitude towards risk constitutes a major determinant of how individual behavior may respond to a change in the standard deviation of life-length. Although this insight is not very surprising in itself, our relatively simple model allows us to derive several quite strong results. In a first best framework, where a benevolent government exercises perfect control over the individuals' consumption, labor supply, and retirement decision, the results imply that a decrease in the standard deviation of life-length leads to (i) an increase in the optimal retirement age, (ii) a decrease in the number of work hours per period spent working, and (iii) an increase in the total number of hour of work over the life-cycle, if the preferences for "the number of years spent in retirement" are characterized by constant or decreasing absolute risk aversion. The same qualitative result holds in a market economy without any public sector intervention at all. The intuition is that reduced uncertainty about the length of life gives an incentive to postpone retirement, because the future benefits associated with retirement are less uncertain than before. In addition, we show that both an actuarially fair pension system and an inheritance tax combined with redistribution can be designed to achieve the socially optimal resource allocation. Under an actuarially fair pension policy, a decrease in the standard deviation of the length of life also leads to a simultaneous decrease in the contribution rate and increase in the pension benefit per period spent in retirement.

Appendix

Comparative Statics of the Model in Section 3

Differentiate equation system (5a) and (5b) to derive the following determinant of the Hessian matrix:

$$\Lambda = \left(U''(c)\frac{(Rw)^2}{\bar{T}} + R\Phi''(H-h) \right)\left(U''(c)\frac{(Rh)^2}{\bar{T}} + EP''(T-R) \right) - \left(U''(c)\frac{w^2 Rh}{\bar{T}} \right)^2 > 0 .$$

The comparative statics become

$$\frac{\partial R}{\partial w} = \frac{-R\Phi''(H-h)\left(U'(c)h + U''(c)wh\frac{Rh}{\bar{T}} \right)}{\Lambda} \tag{A1}$$

$$\frac{\partial h}{\partial w} = \frac{-\left(U'(c)R + U''(c)wR\frac{Rh}{\bar{T}} \right)EP''(T-R)}{\Lambda} \tag{A2}$$

$$\frac{\partial R}{\partial \bar{T}} = \frac{1}{\Lambda}\left(R\Phi''(H-h)U''(C)wh\frac{Rh}{\bar{T}^2} + \left(U''(C)\frac{(Rw)^2}{\bar{T}} + R\Phi''(H-h) \right) E[P''(T-R)] \right) > 0 \tag{A3}$$

$$\frac{\partial h}{\partial \bar{T}} = \frac{E[P''(T-R)]U''(C)\frac{w^2Rh}{\bar{T}}\left(\frac{R}{\bar{T}}-1\right)}{\Lambda} < 0 \tag{A4}$$

$$\begin{aligned}\frac{\partial (hR)}{\partial \bar{T}} &= \frac{\partial h}{\partial \bar{T}}R + \frac{\partial R}{\partial \bar{T}}h \\ &= \frac{1}{\Lambda}\left(RE[P''(T-R)]U''(C)wR\frac{Rwh}{\bar{T}^2} + R\Phi''(H-h)\left(U''(C)wh\frac{Rwh}{\bar{T}^2} + E[P''(T-R)] \right)h \right) > 0\end{aligned} \tag{A5}$$

$$\frac{\partial R}{\partial \gamma} = \frac{\left(U''(C)\frac{(Rw)^2}{\bar{T}} + R\Phi''(H-h) \right)E[\varepsilon P''(T-R)]}{\Lambda} \tag{A6}$$

$$\frac{\partial R}{\partial \gamma} = \frac{-E[\varepsilon P''(T-R)]U''(C)\frac{w^2Rh}{\bar{T}}}{\Lambda} \tag{A7}$$

$$\frac{\partial (hR)}{\partial \gamma} = \frac{\partial h}{\partial \gamma}R + \frac{\partial R}{\partial \gamma}h = \frac{R\Phi''(H-h)E[\varepsilon P''(T-R)]h}{\Lambda}. \tag{A8}$$

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