Public Goods and Optimal Paternalism under Present-Biased Preferences[1]

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Abstract:
This paper concerns the provision of a state-variable public good in a two-type model under present-biased consumer preferences. The preference for immediate gratification facing the high-ability type weakens the incentive to adjust public provision in response to the self-selection constraint.

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Introduction

Over the last decades, numerous studies have reported strong evidence suggesting that people and animals have "present-biased" preferences, i.e. a tendency to give less weight to the future welfare consequences of today's actions than would be optimal for the individual himself/herself in a longer time-perspective (see, e.g., Thaler, 1981; Mazur, 1987; Kirby, 1997; Viscusi, Huber and Bell, 2008; Brown, Chua and Camerer, 2009). Present-biased preferences might be exemplified by quasi-hyperbolic discounting, where the individual, at any time t, attaches a higher utility discount rate to tradeoffs between periods t and $t+1$ than to similar tradeoffs in the more distant future. Viscusi, Huber and Bell (2008) have studied discounting of the benefits attached to a public good, exemplified by water quality. Based on a representative U.S. sample of 2,914 respondents, they estimate the "quasi-hyperbolic discounting parameter" (referred to as "β" below) to be in the interval 0.48-0.61. This suggests that the weight given to benefits in period $t+1$, relative to benefits in period t, is roughly half of the weight that consumers in period t give to benefits in period $t+2$ relative to benefits in period $t+1$.

The purpose of this short paper is to examine how a paternalistic government would modify the policy rule for public good provision in response to quasi-hyperbolic discounting. We focus on a state-variable public good, as many real world public goods, such as, e.g., different aspects of environmental quality, have this particular character. Our study is based on an overlapping generations (OLG) model with two ability-types, where each individual lives for three periods (the minimum number of periods to address quasi-hyperbolic discounting). The government is assumed to carry out redistribution under asymmetric information by means of nonlinear labor and capital income taxation as well as provide the state-variable public good referred to above. Therefore, our concern will be to study the supplemental role of public provision when the income taxes are optimal.

The Model and Main Results

Consider an OLG economy where each consumer lives for three periods; works in the first two and becomes a pensioner in the third. Each generation consists of two consumer-types: a low-ability type earning wage rate w_t^l and a high-ability type earning wage rate $w_t^h > w_t^l$ in period t. The instantaneous utility faced by ability-type i of age a in period t is written

$$u_{a,t}^i = u(c_{a,t}^i, z_{a,t}^i, G_t),$$

where c denotes consumption of a numeraire good, z leisure and G the public

good. The age indicator, a, takes the value 0, 1 and 2, if the consumer is young, middle-aged and old, respectively. When young and middle-aged, leisure is given by a time endowment less the hours of work, i.e. $z^i_{0,t} = H - \ell^i_{0,t}$ and $z^i_{1,t+1} = H - \ell^i_{1,t+1}$, whereas all time is spent on leisure when old, so $z^i_{2,t+2} = H$, for $i = l,h$. The intertemporal objective of ability-type i of generation t is given by

$$U^i_{0,t} = u^i_{0,t} + \beta^i \sum_{j=1}^{2} \Theta_j u^i_{j,t+j},$$

where $\Theta^t = 1/(1+\theta)^t$ is a conventional exponential discount factor with utility discount rate θ (the same for everybody), whereas $\beta^i \in (0,1)$ is a type-specific and time-inconsistent preference for immediate gratification.[4]

Let s denote saving and r the interest rate. We abstract from bequests, meaning that the intial wealth of each consumer is zero. The budget constraint faced by ability-type i of generation t can then be written as follows;

$$\begin{aligned} w^i_{0,t}\ell^i_{0,t} - T^i_{0,t} - s^i_{0,t} &= c^i_{0,t} \\ s^i_{0,t}[1 + r_{t+1}] + w^i_{1,t+1}\ell^i_{1,t+1} - T^i_{1,t+1} - s^i_{1,t+1} &= c^i_{1,t+1} \\ s^i_{1,t+1}[1 + r_{t+2}] - T^i_{2,t+2} &= c^i_{2,t+2} \end{aligned}$$

where the price of the consumption good has been normalized to one. The variables $T^i_{0,t}$, $T^i_{1,t+1}$ and $T^i_{2,t+2}$ represent the income tax payments made when young, middle-aged and old, respectively, which are nonlinear functions of income. Although the optimal use of income taxation will not be examined here, we assume that the income tax system is flexible in the sense of allowing the government to control, the consumption, labor supply and savings behavior of each ability-type.[5]

To simplify, we follow much earlier literature in assuming that output is produced by a linear technology, which is interpreted to mean that the factor prices (wage rates and interest rate) are exogenous.
The public good evolves according to the following difference equation;

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[4]It would add no important insight into the consequences of quasi-hyperbolic discounting if we were to assume that the conventional utility discount factor differs between ability-types.

[5]Aronsson and Sjögren (2009) analyze the optimal use of income and commodity taxation by a paternalistic government when the consumers apply quasi-hyperbolic discounting.

$$G_t = g_t + \delta G_{t-1},$$

where g_t is the incremental provision (or investment in the public good) in period t, while $\delta \in (0,1)$ reflects the depreciation factor.

Turning to public policy, our concern is to analyze the optimal provision of the state-variable public good when decided upon by a paternalistic government; therefore, we assume that $\beta^l = \beta^h = 1$ from the point of view of the government.[6] The objective of the government is represented by a utilitarian social welfare function. The contribution of ability-type i of generation t to this social welfare function becomes

$$V^i_{0,t} = u^i_{0,t} + \sum_{j=1}^{2} \Theta^j u^i_{j,t+j},$$

and the social welfare function is written as

$$W = \sum_t \sum_i \Theta^t V^i_{0,t}.$$

We will later compare the policy rule for public provision following this paternalistic approach with the policy rule that would be chosen by a welfarist government (that respects the individual preferences for immediate gratification).

The informational assumptions are conventional: the government can observe labor and capital income, whereas ability is private information. We focus on the "normal case", where the government attempts to redistribute from the high-ability to the low-ability type. As a consequence, the government must prevent the high-ability type from becoming a mimicker. This can be formalized by introducing a self-selection constraint

$$U^h_{0,t} = u^h_{0,t} + \beta^h \sum_{j=1}^{2} \Theta^j u^h_{j,t+j} \geq \widehat{U}^h_{0,t} = \hat{u}^h_{0,t} + \beta^h \sum_{j=1}^{2} \Theta^j \hat{u}^h_{j,t+j},$$

where $\widehat{U}^h_{0,t}$ denotes the utility of the mimicker. We assume that an individual who reveals himself/herself to be a high-ability type when young cannot credibly pretend to be a low-ability type when middle-aged, which means that the decision of whether or not to become a mimicker is taken by the young high-ability type. The mimicker faces the same income-consumption combinations as the low-ability type; however, as the

[6]This assumption is in line with earlier comparable literature on optimal paternalism; see, e.g., O'Donoghue and Rabin (2003, 2006) and Aronsson and Thunström (2008).

mimicker is more productive, he/she spends more time on leisure than the low-ability type.

For all t, the resource constraint is written

$$\sum_i [w^i_{0,t}\ell^i_{0,t} + w^i_{1,t}\ell^i_{1,t} - c^i_{0,t} - c^i_{1,t} - c^i_{2,t}] + K_t(1+r_t) - K_{t+1} - p_t g_t = 0,$$

where K_t is the capital stock at the beginning of period t, and p_t is a fixed marginal cost of public provision interpretable as the marginal rate of transformation between the incremental public good and the private consumption good in period t. The decision-problem facing the government is to maximize the social welfare function presented in equation (), subject to the accumulation equation for the public good, the self-selection constraint and the resource constraint given by equations (), () and (), respectively. The Lagrangean corresponding to this optimization problem becomes

$$\begin{aligned} L = W &+ \sum_t \mu_t\{\delta G_{t-1} + g_t - G_t\} \\ &+ \sum_t \gamma_t \left\{ \sum_i [w^i_{0,t}\ell^i_{0,t} + w^i_{1,t}\ell^i_{1,t} - c^i_{0,t} - c^i_{1,t} - c^i_{2,t}] - p_t g_t + K_t(1+r_t) - K_{t+1} \right\} \\ &+ \sum_t \lambda_t \left\{ u^h_{0,t} + \beta^h \sum_{j=1}^{2} \Theta^j u^h_{j,t+j} - \widehat{u}^h_{0,t} - \beta^h \sum_{j=1}^{2} \Theta_j \widehat{u}^h_{j,t+j} \right\} \end{aligned}$$

where μ, γ and λ are Lagrange multipliers. In this second best problem, the decision-variables are $\ell^i_{0,t}$, $c^i_{0,t}$, $\ell^i_{1,t}$, $c^i_{1,t}$, $c^i_{2,t}$ (for $i = 1,2$), g_t and K_t for all t.[7]
Let

$$MRS^i_{a,t} = \frac{\partial u^i_{a,t}/\partial G_t}{\partial u^i_{a,t}/\partial c^i_{a,t}} \text{ and } \widehat{MRS}^h_{a,t} = \frac{\partial \hat{u}^h_{a,t}/\partial G_t}{\partial \hat{u}^h_{a,t}/\partial c^l_{a,t}}$$

denote the marginal rate of substitution between the public good and private

[7]Since the government can control the private consumption and work hours by each ability-type via the tax system, it is convenient to write the second best problem as a direct decision-problem (where the government decides upon private consumption and work hours instead of tax parameters). This approach is standard in the literature on optimal nonlinear taxation. See also earlier literature on optimal income taxation in dynamic economies; e.g., Pirttilä and Tuomala (2001) and Aronsson and Johansson-Stenman (2010).

consumption faced by ability-type i of age a in period t and the corresponding marginal rate of substitution faced by the mimicker, respectively. We can then present our main result as follows;[8]

Proposition 1 *When the consumers have present-biased preferences, and the labor and capital income taxes are optimal, the policy rule for the state-variable public good is given by*

$$p_t = \sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \delta^{\tau} SMBG_{\tau}$$

where

$$SMBG_t = \sum_i \sum_a MRS_{a,t}^i + \frac{\lambda_t}{\gamma_t} \frac{\partial \hat{u}_{0,t}^h}{\partial c_{0,t}^l} \left[MRS_{0,t}^l - \widehat{MRS}_{0,t}^h \right] + \beta^h \Theta \frac{\lambda_{t-1}}{\gamma_t} \frac{\partial \hat{u}_{1,t}^h}{\partial c_{1,t}^l} \left[MRS_{1,t}^l - \widehat{MRS}_{1,t}^h \right].$$

Equation () means that the marginal rate of transformation between the public good and the private consumption good in period t, p_t, should equal the sum of social marginal benefits that this investment gives rise to over time, which reflect the marginal willingness to pay for the public good by the consumers as well as effects via the self-selection constraint. This corresponds to the results derived by Pirttilä and Tuomala (2001), yet with the exception that the self-control problem to be discussed below was absent in their study. Note that the instantaneous social marginal benefit in period t, $SMBG_t$, reflects the marginal willingness to pay by all three age-groups, i.e. the young, middle-aged and old, respectively, in period t. Also, the future marginal benefits of an incremental contribution to the public good in period t are not discounted directly by the utility discount rate; instead, the quotient of present value shadow prices attached to the government's budget constraint, $\gamma_{t+\tau}/\gamma_t$, serves this purpose. By using the first order condition for the capital stock in subsequent periods, we have $\gamma_{t+\tau}/\gamma_t = 1/[(1+r_t)(1+r_{t+1})\ldots(1+r_{t+\tau})]$.

Straight forward calculations show that Proposition 1 continues to apply if the paternalistic government is replaced by a welfarist government, whose objective is

[8]Note that Proposition 1 applies irrespective of whether the consumers are naive (erroneously expect not to suffer from the self-control problem in future periods) or sophisticated (in which case the consumer implements a plan that his/her future selves will follow). For a more thorough discussion of naivety versus sophistication, see, e.g., O'Donoghue and Rabin (1999).

written as $\breve{W} = \sum_t \sum_i \Theta^t U^i_{0,t}$ where $U^i_{0,t}$ is given by equation (2). In other words, equations (12) and (13) also reflect the optimal policy rule for public good provision faced by a welfarist government. This result can be understood by observing that the sum of marginal rates of substitution included in the expression for $SMBG_t$ in equation (13) reflects within-period tradeoffs between public and private consumption, which are not directly affected by β. The intuition is that the paternalistic government uses the capital income tax to internalize the intertemporal externality that each young consumer imposes on his/her future selves. As a consequence, there is no need for the paternalistic government to use public provision as an indirect instrument to affect the incentives to save faced by the consumers, which explains why the optimal policy rule for public provision derived for a paternalistic government does not differ from the policy rule a welfarist government would use. However, this does not mean that the level of public provision implemented by a paternalistic government coincides with the level implemented by a welfarist government; instead, as the optimal consumption-saving tradeoff faced by a paternalistic government differs from that faced by a welfarist government, so will the optimal time path for the public good.

We can see from equation () that the self-control problem facing the consumers directly affects the policy rule for the public good via the self-selection constraint; more precisely via the contribution to this constraint by the middle-aged generation.[9] Two interesting observations follow immediately from equation (). First, the preference for immediate gratification weakens the contribution that the self-selection constraint has on the policy rule for public provision. In a sense, therefore, quasi-hyperbolic discounting brings us closer to (a dynamic analogue to) the first best policy rule that would apply without asymmetric information. Second, it is only the preference for immediate gratification facing the high-ability type, β^h, that affects the policy rule directly; there is no corresponding effect of β^l. We summarize these observations in the following corollary to Proposition 1;

Corollary 1. *All other things equal, the preference for immediate gratification faced by the high-ability type, $\beta^h \in (0,1)$, weakens the policy incentive associated with the self-selection constraint. The smaller β^h, ceteris paribus, the weaker will be the incentive created by the second row of equation () to overprovide (underprovide) the public good relative to the Samuelson rule if $MRS^l_{1,t} >$ ($<$) $\widehat{MRS}^h_{1,t}$. There is no corresponding effect of the preference for immediate gratification faced by the low-ability type.*

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[9] Since $MRS^l_{2,t} = \widehat{MRS}^h_{2,t}$, the corresponding effect for the old generation vanishes. The reason is that the old consumers spend all available time on leisure regardless of ability.

When the young high-ability type decides whether or not to become a mimicker, he/she attaches less weight to the future utility consequences of today's actions than he/she would have done in the absence of the self-control problem (as the utilities facing the young consumer's middle-aged and old selves are multiplied by $\beta^h < 1$). As a consequence, the welfare contribution of public provision that goes via $MRS^l_{1,t} - \widehat{MRS}^h_{1,t}$ is only a fraction of the corresponding effect that would follow without the preference for immediate gratification, which explains the first part of Corollary 1. The final part follows because the preference for immediate gratification facing the low-ability type does not directly affect the self-selection constraint. In other words, self-control problems facing mimicked agents (who are not potential mimickers themselves) will not modify the policy rule for the public good.

In the special case where $\delta = 0$, in which the public good becomes a flow variable, equation () reduces to read $p_t = SMBG_t$, meaning that the forward-looking benefit measure reduces to a static measure. The qualitative effects of quasi-hyperbolic discounting will, nevertheless, remain as in Corollary 1: β^h still affects the policy rule via the self-selection constraint faced by the middle-aged, and β^l does not modify the policy rule for a flow-variable public good.

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